New Models and Algorithm for Throughput Maximization in Broadcast Scheduling

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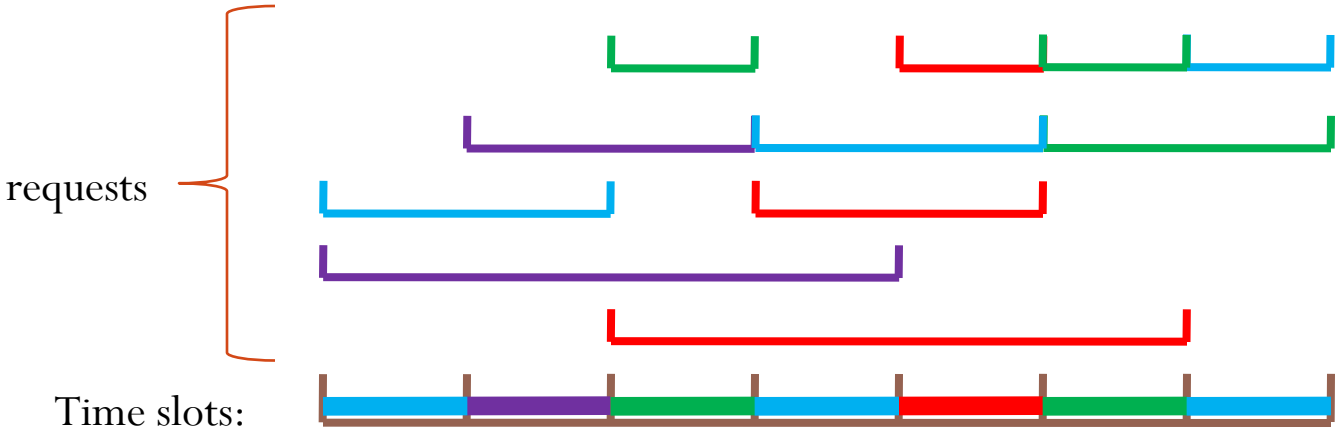
Broadcast Scheduling

Problem definition:

- Given a set of pages $P=\{p_1,p_2 \Sigma p_n\}\Sigma$
- Time is slotted, ¢ Γ ϑ м Σ н Σ X Σ ¢ Y
- Each client sends a request *r* for page *p, with* release time a_r and deadline d_r
- The server broadcasts one page *p* in a time slot *t*, and all requests *r* of page *p* with $t2[a_r, d_r]$ can be satisfied

Broadcast Scheduling

- Example:



Broadcast Scheduling

- Traditional objectives.
  - Hard deadlines:
    - Throughput maximization (MAX-THP)
    - X
  - No deadlines:
    - Minimizing the max response time.
    - Minimizing the flow time (i.e., avg. response time).
    - X Φ
  - NP-hardness [Chang et al. 08].

Motivation

- Each client request the reading of some sensor at some time. The server can probe one sensor in a time slot.
  - A client requests the temperature reading at 5:30PM. She may be satisfied with a reading at 5:33PM. A reading at 5:40PM may be still useful, but not as much. But a reading in 6:00PM is useless.
- Traditional objectives are not sufficient in this example.
  - Minimizing response time ignores deadlines.
  - Minimizing throughput ignores the latency of satisfied requests.
- We capture this in two approaches.
  - A general time-dependent profit function.
  - Tradeoff between completeness and latency.

Profit Maximization

- A generalization of throughput maximization: Profit Maximization (MAX-PFT)
  - A time ɕ dependent profit function $g_r(t)$ for each request r.
  - If a request is satisfied multiple times, we take the maximum one.
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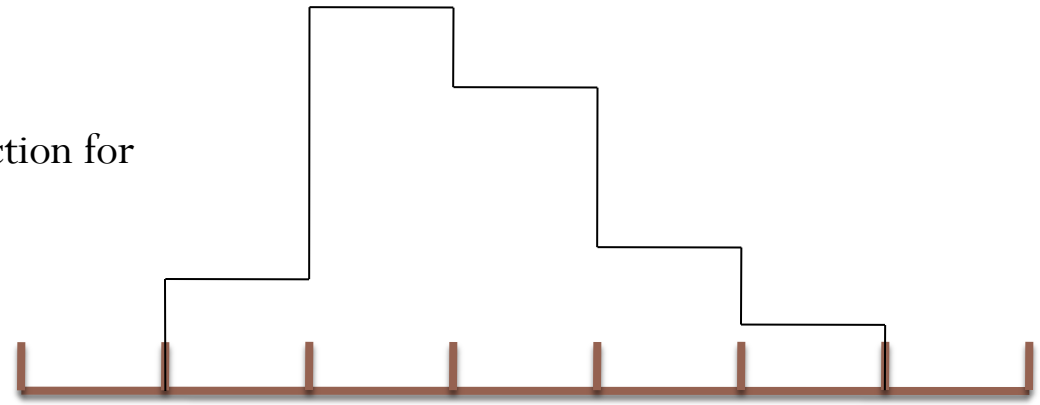
The profit function for a request:



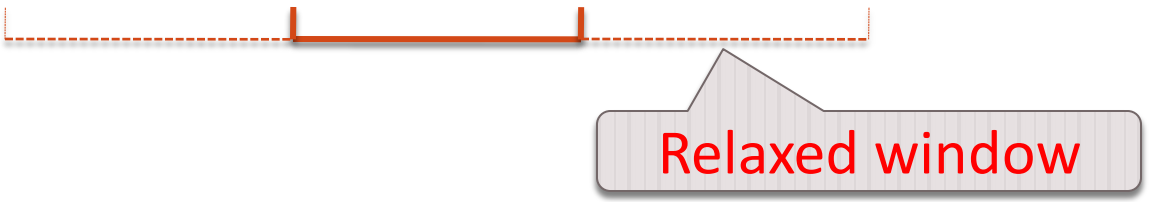
Our Results

- Offline setting.
  - A (1-1/e)-approximation for MAX-PFT.
  - A 3/4-approximation for MAX-PFT when the profit functions are unimodal.
    - MAX-THP offline: A 3/4 -approximation ώ D Ι· γ Ř Κ ƛ . Ŝ ű Ι· f Φ Ω

- Online settings.
  - An *s*-speed (*1+1/s*) ϙompetitive algorithm for MAX-PFT.
    - MAX-THP online: A 1/2-competitive algorithm ώ Υ ƛ Υ Ŝ.ű Ι· f Φ

Our Results

- Minimizing latency subject to completeness requirement.
  - A (3/4, 1)-approximation for the (completeness, latency) pair.
  - Note that both ratios are *in expectation*.
- Throughput Maximization with Relaxed Time Windows.
  - Suppose there is a fractional solution that satisfies all requests. We can find a schedule in polynomial time such that each request can be satisfied by right (or left) shifting the window by at most its length.

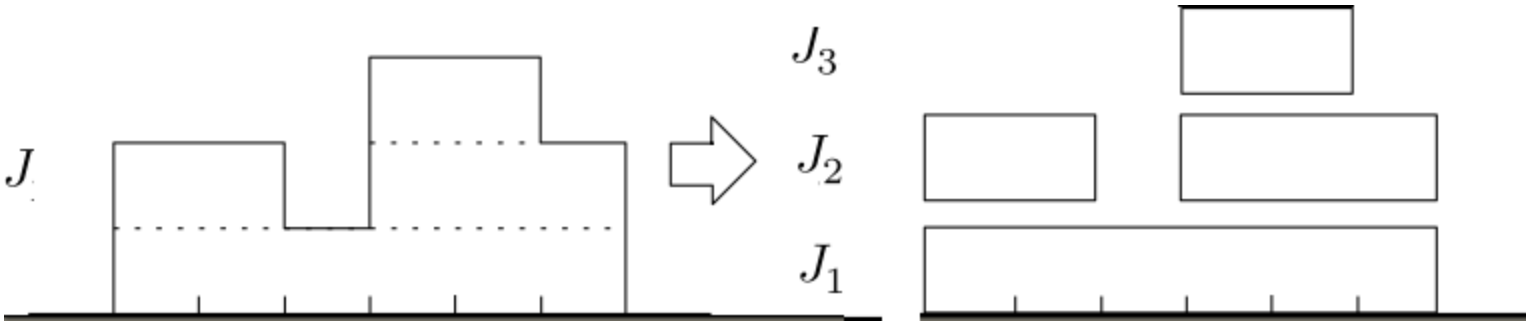


Our Results

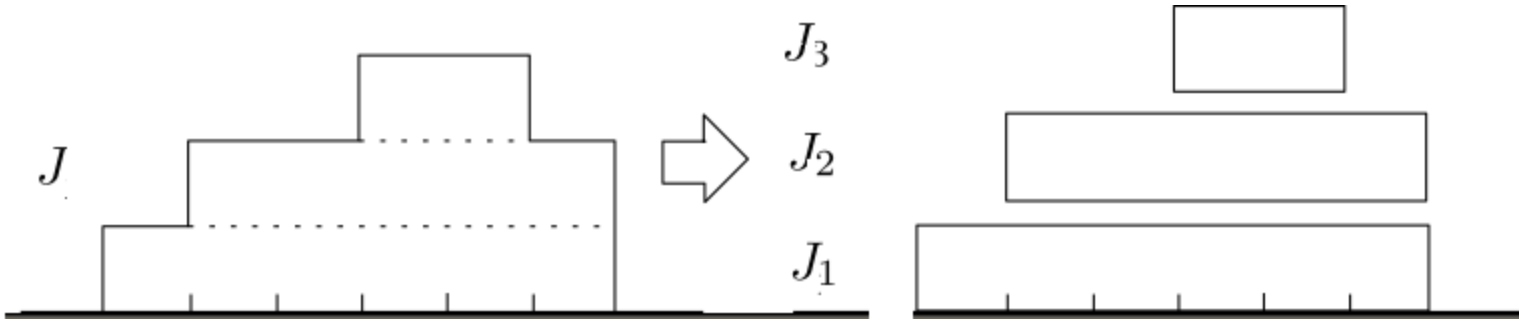
- Offline MAX-THP:
  - 2-speed 1-approximation.
    - Such a result was known only if all request can be scheduled in a fractional solution [Chang et al. 08].
    - This directly implies a 2-approximation for MAX-THP.
- Minimizing the max response time
  - A $(2-\frac{1}{B})$-lower bound for randomized algorithms in the oblivious adversary model.
    - The same bound was only known for deterministic algorithm [Bartal et al. 00, Chang et al. 08].
    - Note that FIFO is 2-competitive [Bartal et al. 00, Chang et al. 08, Chekuri et al. 09].

Offline ςProfit Maximization

- The slicing trick: Convert MAX-PFT to weighted MAX-THP



A general profit function



A unimodal profit function

Offline ςProfit Maximization

THM: A 3/4 ςapproximation for MAX-PFT when the profit functions are unimodal.

Proof: The slicing trick and the 3/4-approximation for weighted MAX-THP.

THM: A (1-1/e) ςapproximation for MAX-PFT with general profit functions.

Proof 1: A simple independent rounding schema.

Offline ςProfit Maximization

Proof 2:

(submodular maximization subject to a matroid constraint)

- f: 2^N ПЂ w ƛ ă l submodular function if

$$f(A+x)-f(A) \cdot \quad f(B+x)-f(B) \quad \forall B \subseteq A, x \in N$$

- Let N be *{(p,t)}*$_{p,t}$. The set of feasible solutions is a partition matroid.
- Let *Profit(S)* be the profit obtained by schedule *S* (*S*μ*N*). *Profit(.)* is submodular.
- Submodular function maximization subject to a matroid constraint: (1-1/e)-approximation [Calinescu Ŝ ű l· ƒ Ф Ω Ω л у Vondrak Σ Chekuri Ŝ ű l·. ƒ Ф Ω м л в

Online ςProfit Maximization

- Maximum Additional Profit First (MAPF):
  - At any time *t,* broadcast *s* pages which give the maximum additional profits.

THM: MAPF is an *s*-speed (*1+1/s*)-competitive online algorithm for MAX-PFT.

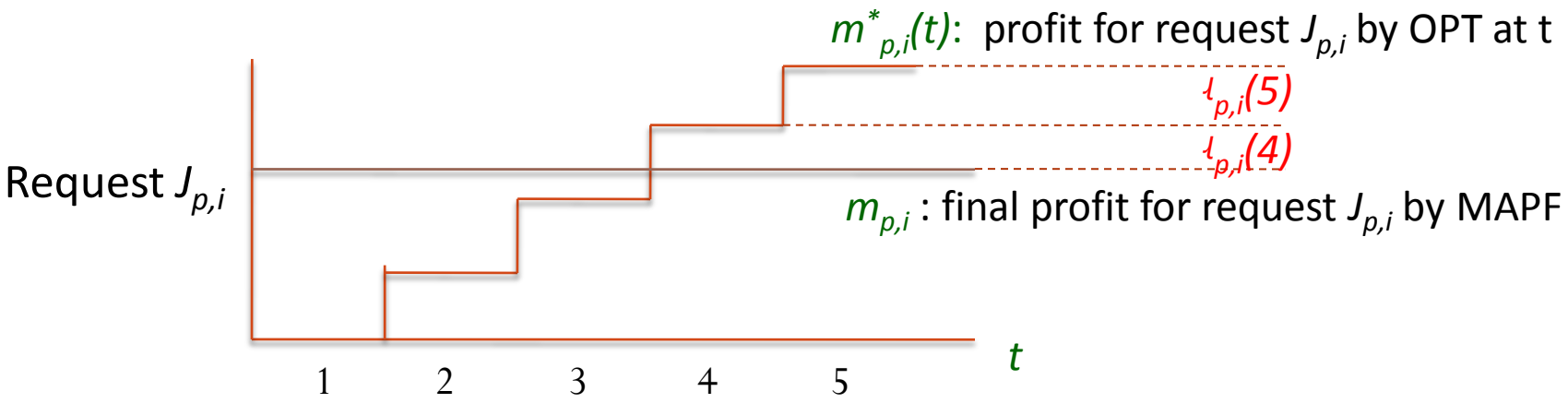
The analysis is tight.

THM: For any *ь>0* and *ǎ ϰ,ϰ*, MAPF is not *s*-speed *(1+1/s-ь)*-competitive.

Online ςProfit Maximization

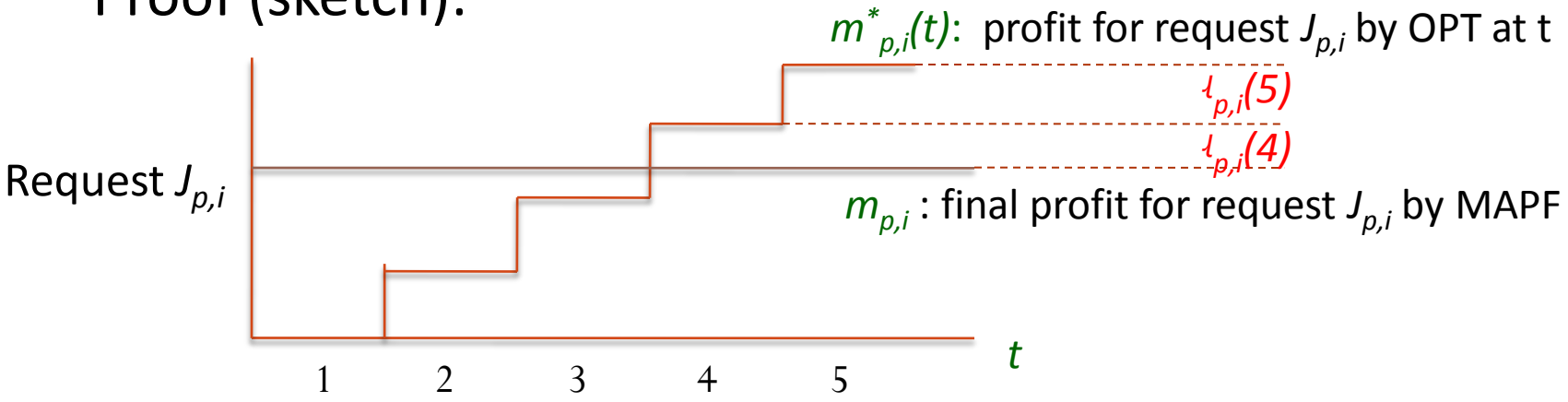
Proof (sketch):

- the increase of the so-far-gained profit by OPT over the final profit by MAPF.



Online ςProfit Maximization

Proof (sketch):



ƙ ó ú́ ú́ Γ̍ $_{p,i}$ $_{p,i}$ (t).

h t ¢ Ж ą ƙ(t) C t -- by definition of ƙ Φ

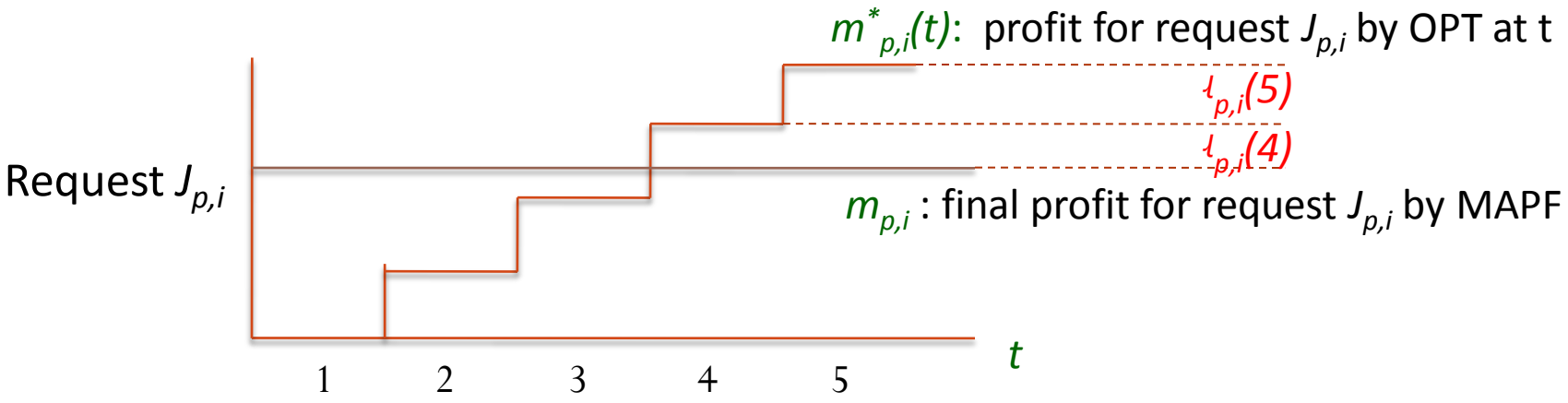
Ж a ! C (1/s) Ъ $_{p,i}$ ($m_{p,i}$(t)-$m_{p,i}$(t-1)) --next slides.

Ж ó м Ъ м к ă ú a ! C t

Additional profit obtained by MAFP

Online ςProfit Maximization

Proof (sketch):



It suffices to show ƙó ǘ ú ʅ$_{p,i}$ ʅ$_{p,i}$(t) ʌ ǎ$_{p,i}$(t-1))

- Assume OPT broadcast *q* at time *t* and ƙ(t)>0.
- We can show MAPF does not broadcast *q*. O.w. ƙ(t)=0.
- ƙó ǘ ʅ$_i$(ú$_{q,i}$(t)-$m_{q,i}$(t-1)).
- ($m_{q,i}$(t)-$m_{q,i}$(t-1)) Ʌ ʅ$_{p,i}$(ó$_{p,i}$(t)-$m_{p,i}$(t-1)) if MAPF broadcast *p*.

Open Problems

- Is it possible to get a 4/3-speed 1-approximation for MAX-THP. Note this would imply a 3/4-approximation for MAX- ¢ I t ó Y l· ű Ŏ Ƙ ƛ ƴ Ǝ ű Ƙ Ŝ ō Ƨ
- Derandomizing the (3/4,1)-approximation for the (completeness, latency) pair.
- A better understanding of the completeness-latency tradeoff.

Thanks

Offline ςProfit Maximization

Proof 1: (LP rounding)

$Y_p^{(t)}=1$: The server broadcasts p at time t.

$X_{p,i}=1$: The ith request of p is satisfied.

$$\text{maximize} \quad \sum_{p;i} w_{p;i} X_{p;i}$$

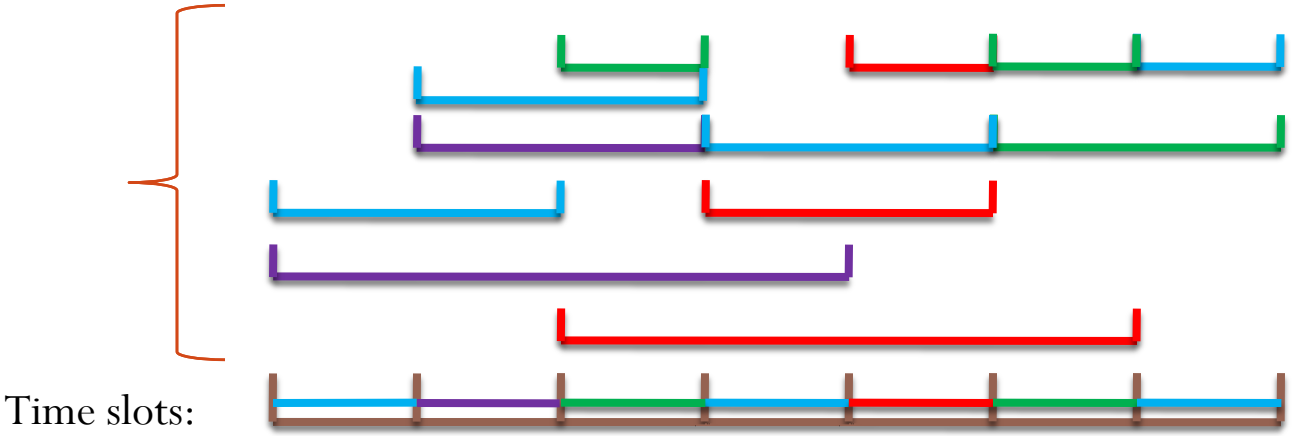
$$\text{subject to} \quad Y_p^{(t)} \geq X_{p;i} \ \forall p,t,$$

$$\sum_{t \in T_{p;i}} Y_p^{(t)} \leq 1, \ \forall t,$$

$$\sum_{p} X_{p;i} \in \{0,1\}, \forall p,t, \quad Y_p^{(t)} \in \{0$$

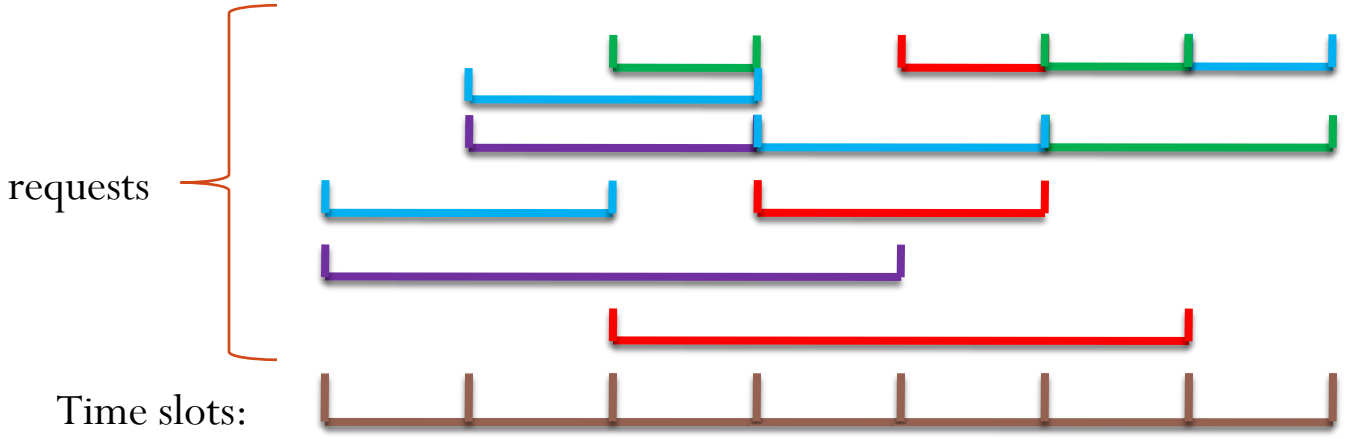
Throughput Maximization with Relaxed Time Windows

- The fractional solution corresponds to a flow.
- There is integral flow.



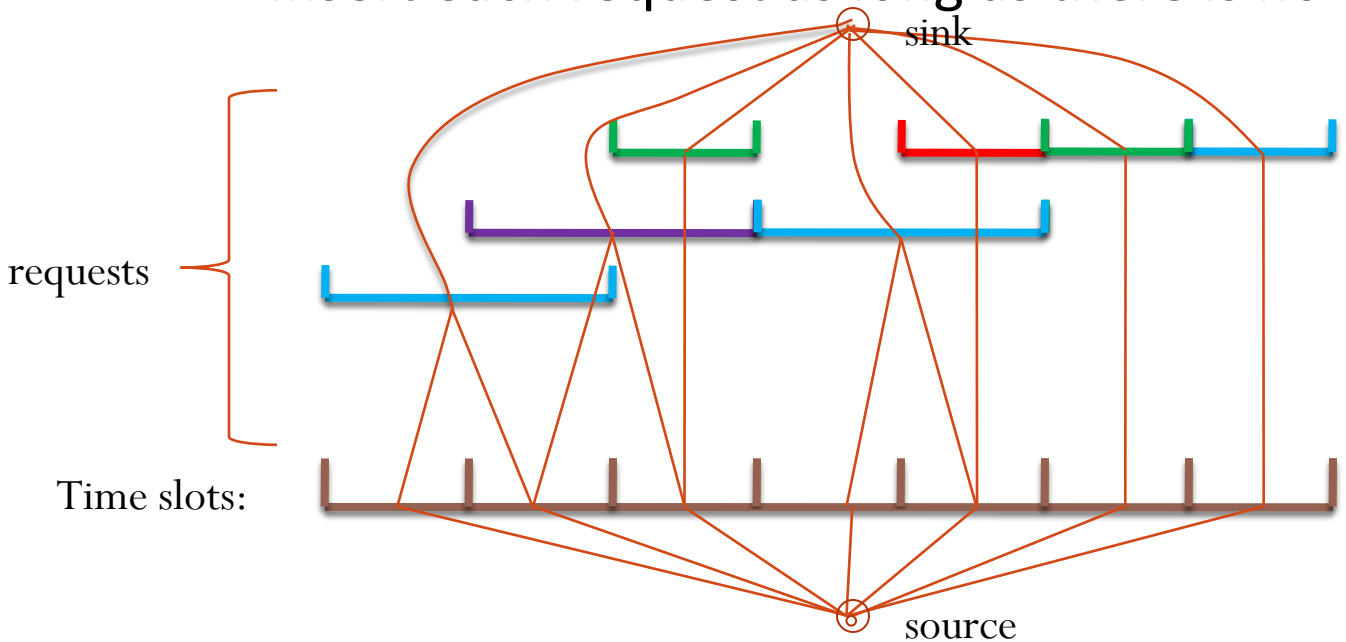
Throughput Maximization with Relaxed Time Windows

- Suppose there is a fractional solution that satisfies all requests.



Throughput Maximization with Relaxed Time Windows

- For each page
  - Order the requests for page p in non-decreasing window length.
  - Insert each request as long as there is no overlap



  - The fractional solution corresponds to a flow.