Generalized Machine Activation Problem

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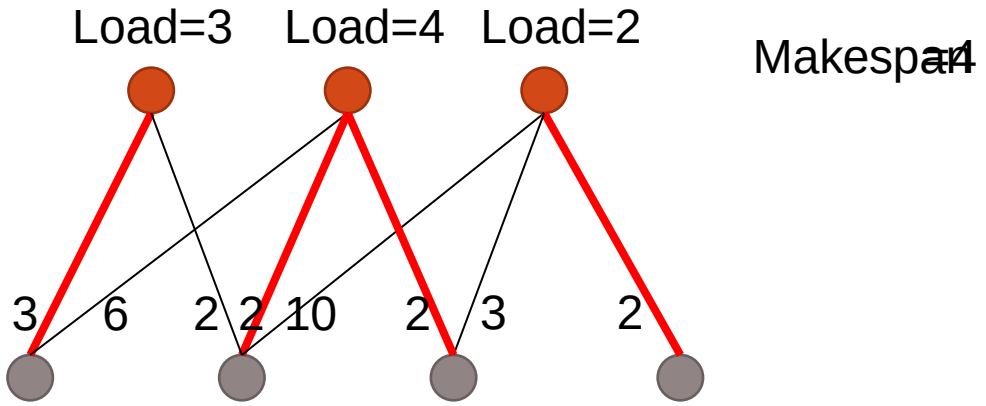
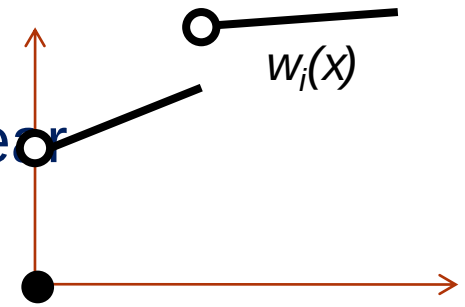
Problem Definition

- Unrelated Machine Scheduling:
  - M: the set of machines
  - J: the set of jobs
  - p_{ij}: processing time of job j on machine i
  - Goal: find an assignment s.t. the makespan is minimized



Problem Definition

- Generalized Machine Activation (GMA):
  - Machine Activation Cost:
    - $w_i(x)$: activation cost function of machine i
      - --- A function of the load of machine i
      - --- Non-decreasing and piecewise linear
      - --- Left-Continuous



  - Assignment Cost
    - a_{ij}: the cost of assigning job j to machine i
  - Objective
    - Find an assignment such that the total cost (i.e., machi
      activation cost plus assignment cost) is minimized

Problem Definition

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  - Machine Activation Problem ώ Y Ƙ dz f f Ŝ NJ Σ [ƛ Σ { l· Ƙ l·
    - The activation cost for each machine is fixed; We require the makespan is at most *T* and minimize the total cost
    - $w_i(x)=w_i$ for $0<x<=T$ and $w_i(x)=\kappa_i$ for $x>T$
  - Universal Facility Location [Hajiaghayi,Mahdian,Mirrokni Ω ф ф ϐ [Mahdian Σ t l· f Ω л o ϐ
    - $p_{ij}=1$ for all i,j, i.e., the activation cost (i.e., facility opening cost) of machine *i* is an increasing function of the number of jobs assigned to *i*
  - Generalized Submodular Covering [Bar- L f l· ƴ Σ Y ƨ NJ ű ă l·
    - GSC generalizes the average cost center problem, the fault tolera facility location problem and the capacitated facility location problem.

Our Results

THM: There is a polynomial time algorithm that finds a fractional assignment such that $1-\beta$ jobs are fractionally satisfied and the cost is at most $\ln(n/\beta)+1$ times the optimal solution.

- Machine Activation Problem
  - Bicriteria approximation: (makespan, total cost)
  - Previous results:
    - $(2+\beta, 2\ln(2n/\beta)+5)$ ώ C f Ŝ ƛ ă, $(3+\beta, (1/2)\ln(n)+1)$ ώ Y [{ Ω
    - No assignment cost: $(2+\beta, \ln(n/\beta)+1)$ ώ C f Ŝ ƛ ă, $(2, \ln(n)+1)$ ώ Y [{ Ω м л б
  - Our results
    - $(2, (1+o(1))\ln(n))$

Our Results

- Universal Facility Location
  - Previous results:
    - Metric: Constant approximations [Mahdian
    - Non-metric: Open [Hajiaghayi,Mahdian,Mirrokni
  - Our results
    - Non-metric: *(ln(n)+1)*-approximation
- Generalized Submodular Covering
  - Previous results:
    - *O(ln nM)*-approximation where *M* is the largest integer in the instance
  - Our results:
    - *ln(D)*-approximation where *D* is the total demand

Our Results

Machine Activation with Linear Constraints

- Each machine has a fixed activation cost
- For each machine, the set of jobs assigned to it must satisfy a set of *d* linear constraints

$$\sum_{j \in J} p_{ijk} x_{ij} \le T_{ik} \quad i \in M; k = 1; 2; \ldots; d$$

E.g., makespan

- THM: For any $\varepsilon > 0$, there is a poly-time algorithm that returns an integral schedule X,Y such that
  1. $\sum_{j \in J} p_{ijk} X_{ij} \le (2d + \varepsilon) T_{ik}$ for each i and $1 \le k \le d$;
  2. $\mathbb{E}[\sum_{i \in M} c_i Y_i] \le O(\frac{1}{\varepsilon} \log n) \sum_{i \in M} c_i y_i$.
- This matches the previous bound for $d = 1$ [KLS10]

Outline

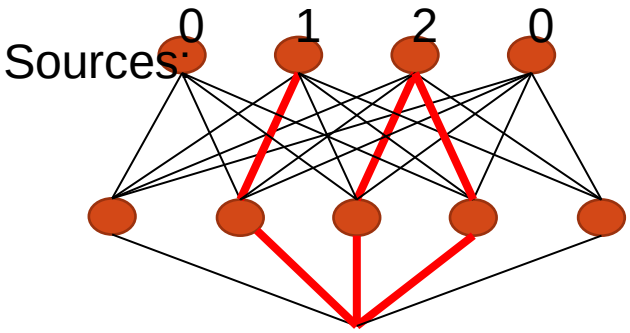
- Greedy for Universal Facility Location
- Greedy for Generalized Machine Activation
- Final Remarks

Greedy for UFL

- A set of facilities (machines) and clients (jobs)
- Facility opening cost $w_i(u_i)$ which is a non-decreasing function of the load of facility i (load = #clients assigned to it)
- Assignment cost a_{ij}

- $\boldsymbol{u}$: the load vector
- $\pi(\boldsymbol{u})$: min. assignment cost under load vector $\boldsymbol{u}$
- $C(\boldsymbol{u}) = \sum_i w_i(u_i) + \pi(\boldsymbol{u})$
  - -- $\pi(u)$ can be computed via a min-cost flow

u=<0,1,2,0>



Greedy for UFL

- $\mathbf{u}$: the load vector
- $C(\mathbf{u}) = \sum_i w_i(u_i) + \phi(\mathbf{u})$
- $\mathbf{e}_i$ [illegible]

  The ith entry
- GREEDY-UFL

  Repeat

  -- choose the machine i and integer $k>0$ such that

$$\rho(\mathbf{u}, i, k) = \frac{C(\mathbf{u}+k\mathbf{e}_i) - C(\mathbf{u})}{k}$$

  is minimized.

  Until all jobs are served (i.e., $|\mathbf{u}| = n$)

Greedy for UFL

- Analysis:
  - We would like to show

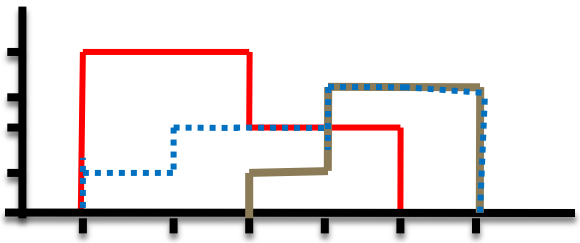
$$\min_{i,k} \frac{\Phi(\mathbf{u}, i, k)}{n_i - |\mathbf{u}|} \cdot \frac{C(\mathbf{u}^*)}{n - |\mathbf{u}|}$$

where $\mathbf{u}^*$ is the optimal load vector

Lemma: For any load vector $\mathbf{u}$, there exists $\mathbf{a}$ such that

1. $\mathbf{u} \cdot \mathbf{a} \cdot \max(\mathbf{u}, \mathbf{u}^*)$
2. $\Phi(\mathbf{a}) \cdot \Phi(\mathbf{u}^*) + \Phi(\mathbf{u})$
3. $|\mathbf{a}| = n$

$\mathbf{u} = \langle 0; 0; 1; 3; 3 \rangle$
$\mathbf{u}^* = \langle 4; 4; 2; 2; 0 \rangle$
$\mathbf{a} = \langle 1; 2; 2; 3; 3 \rangle$



Greedy for UFL

- Analysis Cont:

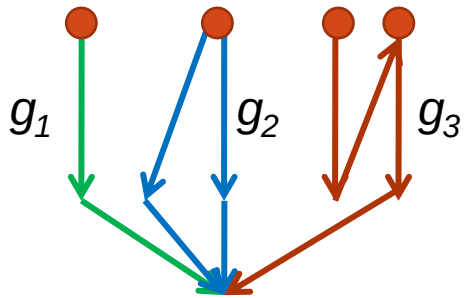
f (or f^e) is the optimal flow corresponding to (or $\mathbf{u}$)

Consider the flow $g = f^e - f$

(1) We can easily show g is a feasible flow in the residual graph w.r.t. f

(2) Apply the conformal path decomposition to g.

(3) Divide the paths into groups $g_1, g_2, \ldots$ base on the sources of the paths (indicated by colors)

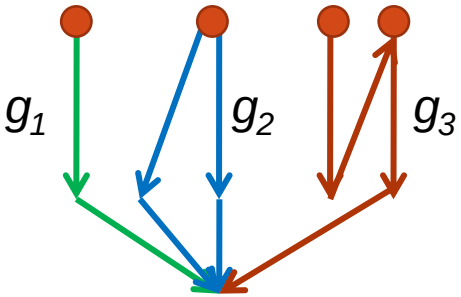


Such a structure is due to the fact that $\mathbf{u} \geq \mathbf{u}$

Greedy for UFL

- Analysis cont.

1. $\mathbf{u} \cdot \mathbf{a} \cdot \max(\mathbf{u}; \mathbf{u}^{\alpha})$
2. $¼(\mathbf{a}) \cdot ¼(\mathbf{u}^{\alpha}) + ¼(\mathbf{u})$



$$\sum_{i} c(g_i) = c(g) = c(f^{e}) - c(f) = ¼(\mathbf{a}) - ¼(\mathbf{u}) \cdot ¼(\mathbf{u}^{\alpha})$$

Lemma (2)

Therefore,

Lemma (1)